Exercise 19

In Exercises 17–24, find the unknown if the solution of each equation is given:

$$\text{If } u(x) = \sin x \text{ is a solution of } u(x) = f(x) + \sin x - \int_0^{\frac{\pi}{2}} xu(t)\, dt, \text{ find } f(x)$$

Solution

Substitute the solution into both sides of the equation.

$$\sin x = f(x) + \sin x - \int_0^{\frac{\pi}{2}} x \sin t\, dt$$

Subtract $\sin x$ from both sides and bring x in front of the integral.

$$0 = f(x) - x \int_0^{\frac{\pi}{2}} \sin t\, dt$$
$$= f(x) - x(-\cos t)\Big|_0^{\frac{\pi}{2}}$$
$$= f(x) - x(0 + 1)$$
$$= f(x) - x$$

Therefore,

$$f(x) = x.$$